DATE OF EXAM **Solution**

SUBJECT NAME: Advanced Functional Analysis - FINAL OR MIDTERM Exam: Back Paper 2010-11 - Semester ENTER I or II: M. Math IInd Yr

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1. *Let X, Y be topological vector spaces. Let $T : X \to Y$ be a linear map. Show that T is continuous if and only if it is bounded.*

   Solution: We can find the proof in the book of Walter Rudin, Functional Analysis, Theorem 1.32, page-24.

   $\square$

2. *Let X be a L C T V S. Let $K \subset X$ be a compact set. Show that K is totally bounded.*

   Solution: We can find the proof in the book of Walter Rudin, Functional Analysis, Theorem 3.20 part c, proof, page-72.

   $\square$

3. *Let X and Y be completely metrizable topological vector spaces. Let $T_n : X \to Y$ be a sequence of continuous linear maps such that $\lim\limits_{n\to\infty} T_n(x)$ exists for all $x \in X$. Show that $T(x) = \lim\limits_{n\to\infty} T_n(x)$ is a bounded linear map.*

   Solution: We can find the proof in the book of Walter Rudin, Functional Analysis, Theorem 2.8, page-46.

   $\square$

4. *State and prove the Banach-Alaoglu theorem for topological vector spaces.*

   Solution: We can find the statement and proof in the book of Walter Rudin, Functional Analysis, Theorem 3.15, page-68.

   $\square$

5. *Let K be a compact convex set in a L C T V S, X. Let $F \subset K$ be an extreme, convex, closed set. Show that F has an extreme point of K.*

   Solution: We can find the proof in the book of Walter Rudin, Functional Analysis, Theorem 3.23, proof, page-75. $\square$

6. *Let X be a Banach space and $(\Omega, \sum, \mu)$ a probability space. Let $f : \Omega \to X$ be a strongly μ-measurable function. Show that f in a.e separable valued.*

   Solution: We can find the proof in the book of Joseph Diestel and John Jerry Uhl, Vector Measures. Corollary 3, page-42. $\square$

7. *Let X and f be an question 6. Suppose $\int\limits_{\Omega} ||f(w)||d\mu(w) \leq \infty$. Show that f is Bochner integrable.*

   Solution: We can find the proof in the book of Joseph Diestel and John Jerry Uhl, Vector Measures. Theorem 2, page-45.

   $\square$

8. *Let* $f : [0,1] \to C[0,1]$ *be a measurable function such that inverse image of a Borel set is Borel. Is* f *is strongly measurable ? Justify your answer.*

   Solution: Since $C[0,1]$ is separable, strongly measurable and Borel measurable are equivalent. Therefore f is strongly measurable.

   $\square$

9. *Let* X *be a Banach space and* $f : [0,1] \to X$ *be a Bochner integrable function w.r.t the Lebesgue measure. Show that* $\lim\limits_{\lambda(E)\to 0} \int\limits_E f d\lambda = 0$.

   Solution: We can find the proof in the book of Joseph Diestel and John Jerry Uhl, Vector Measures. Theorem 4 (i), page-46.

   $\square$

10. *State and prove the Pettis measurability theorem.*

    Solution: We can find the statement and proof in the book of Joseph Diestel and John Jerry Uhl, Vector Measures. Theorem 2, page-42.

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